

$$\begin{aligned}
&= \arg \min_{g(n) \in \mathcal{L}} W_1 \left( g^t \Theta(\theta_{1_1}, \theta_{1_2}) g - s^t \Theta(\theta_{1_1}, \theta_{1_2}) g - g^t \Theta(\theta_{1_1}, \theta_{1_2}) s + s^t \Theta(\theta_{1_1}, \theta_{1_2}) s \right) + \\
&\quad \sum_{l=2}^Q W_l g^t \Theta(\theta_{l_1}, \theta_{l_2}) g
\end{aligned} \tag{25}$$

$$\begin{aligned}
&= \arg \min_{g(n) \in \mathcal{L}} \sum_{l=1}^Q W_l g^t \Theta(\theta_{l_1}, \theta_{l_2}) g - \\
&\quad W_1 \left( s^t \Theta(\theta_{1_1}, \theta_{1_2}) g + g^t \Theta(\theta_{1_1}, \theta_{1_2}) s - s^t \Theta(\theta_{1_1}, \theta_{1_2}) s \right)
\end{aligned} \tag{26}$$

In the penultimate line, equation (8) was substituted. To shorten writing, the two quantities  $\theta_{1_1} = k \frac{2\pi}{M}$  and  $\theta_{1_2} = (k+1) \frac{2\pi}{M}$  were introduced. The pulse response of the nominal transmission function is combined in the column vector  $s$ ,

$$[s]_n = -\frac{1}{2} \left( e^{j\frac{\Delta\phi}{2}} h_{k-1}(n) + e^{-j\frac{\Delta\phi}{2}} h_{k+2}(n) \right) \quad \text{for } n = 0, 1, \dots, M-1. \tag{27}$$

The direct minimization via  $g(n)$ , as it is executed in (26) does not make sense for two reasons: first, minimization has to be done over a large number of parameters ( $M$  coefficients) and second, optimization has to take place under the boundary condition  $g(n) \in L$ . It makes more sense to minimize, starting from the coefficient vector  $c$  (19): first, the number of the parameters to be optimized is reduced and second, minimization may occur without secondary conditions as the statement (19) already takes the secondary condition  $g(n) \in L$  into consideration. Substitution of (19) in (26) yields the following:

$$c_{opt} = \arg \min_c \sum_{l=1}^Q W_l c^t H^t \Theta(\theta_{l_1}, \theta_{l_2}) H c - \quad (28)$$

$$W_1 \left( s^t \Theta(\theta_{l_1}, \theta_{l_2}) H c + c^t H^t \Theta(\theta_{l_1}, \theta_{l_2}) s - s^t \Theta(\theta_{l_1}, \theta_{l_2}) s \right) \quad (29)$$

The solution to this optimization problem reads

$$c_{opt} = H^{-1} \left( H^t \sum_{l=1}^Q W_l \Theta(\theta_{l_1}, \theta_{l_2}) \right)^{-1} W_1 H^t \Theta(\theta_{l_1}, \theta_{l_2}) s. \quad (30)$$

Substitution in (19) yields the pulse response of the compensation pulse looked for.

$$g = H c_{opt} = \left( H^t \sum_{l=1}^Q W_l \Theta(\theta_{l_1}, \theta_{l_2}) \right)^{-1} W_1 H^t \Theta(\theta_{l_1}, \theta_{l_2}) s \quad (31)$$

Simulations showed that this pulse only insufficiently meets the required properties. For this reason, the compensation pulse must be allowed to have a length superior to  $M$ . At this point it also makes sense to relinquish the restriction of the distortion-free channel.

It is assumed that the channel possesses a memory length of maximum  $P$ , the pulse response of the channel accordingly having a length of maximum  $P + 1$  taps. For DMT systems, a cyclical

prefix is used which substantially simplifies distortion in the receiver.

With the cyclical prefix, the last  $P$  symbols of a data block are sent first at the beginning of a block, compare **FIG. 15**. Considering the transmission of one single pulse only, it may be written for the transmission sequence  $y(n)$

$$y(n) = \begin{cases} a_{M-P+n} & \text{für } n = 0, 1, \dots, P-1 \\ a_{n-P} & \text{für } n = P, P+1, \dots, N+P-1 \end{cases} \quad (32)$$

Substitute the IDFT for  $a_n$  yields

$$y(n) = \begin{cases} \frac{1}{M} \sum_{k=0}^{M-1} A_k e^{j \frac{2\pi}{M} k(M-P+n)} = \frac{1}{M} \sum_{k=0}^{M-1} A_k e^{j \frac{2\pi}{M} k(n-P)} & \text{für } n = 0, 1, \dots, P-1 \\ \frac{1}{M} \sum_{k=0}^{M-1} A_k e^{j \frac{2\pi}{M} k(n-P)} & \text{für } n = P, P+1, \dots, N+P-1. \end{cases} \quad (33)$$

As can be seen, the case discrimination is abolished on account of the  $M$  periodicity of the IDFT, the transmission sequence reads

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} A_k e^{j \frac{2\pi}{M} k(n-P)} \quad \text{for } n = 0, 1, \dots, N+P-1. \quad (34)$$